

Photonic

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Collaborators

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 - Raksha Singla
 - Lucila Juárez
 - José Samuel Pérez
 - Bernardo Mendoza
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-
- Ed *mohawk2* (many improvements, contributions, suggestions, optimizations, modularization,...)

(Thanks to DGAPA-UNAM IN109822)



What is it?

A package to compute the optical properties of *metamaterials* and *photonic crystals*.



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Module Version: 0.009 [Source](#)

[NAME](#)
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[SYNOPSIS](#)
[DESCRIPTION](#)
[AUTHORS](#)
[ACKNOWLEDGMENTS](#)

[NAME](#) ⓘ

Photonic - A perl package for calculations on photonics and metamaterials.

[VERSION](#) ⓘ

Version 0.009

[SYNOPSIS](#) ⓘ

```
use Photonic::Geometry;  
use Photonic::NonRetarded::EpsTensor;  
  
my $g=Photonic::Geometry->new(B=>$b);  
my $eps=Photonic::Nonretarded::EpsTensor->new(geometry=>$g, nh=>$N);  
my $epsValue=$eps->evaluate($epsA, $epsB);
```

Calculates the dielectric tensor of a metamaterial made up of two materials with dielectric

[DESCRIPTION](#) ⓘ

Set of packages for the calculation of optical properties of metamaterials. The included pa

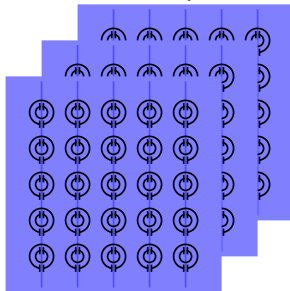
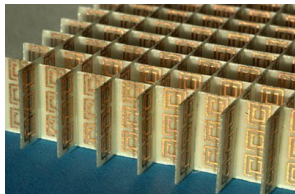
- Public domain
 - Github
 - CPAN
- Using
 - Perl
 - PDL
 - Moo



Metamaterials

Example: split rings

Material made from a mixture of *particles* of ordinary materials, properties usually differ from those of its components.



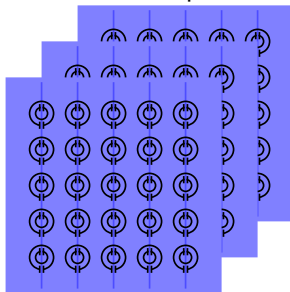
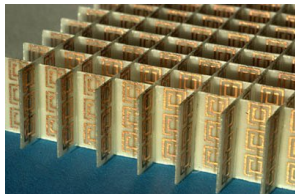
- **LC** like resonances in ϵ , μ .
- Can have $\epsilon < 0$ and $\mu < 0$, but $n^2 = \epsilon\mu > 0$, i.e., real index of refraction, propagation of e.m. waves.
- *Exotic* behavior from mix of ordinary materials, negative index of refraction $n < 0$.



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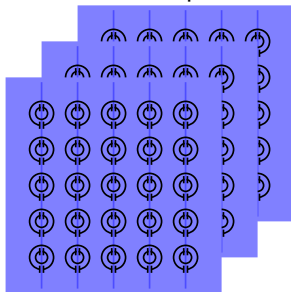
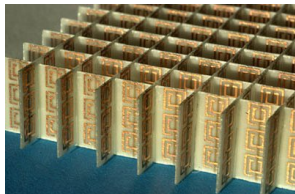
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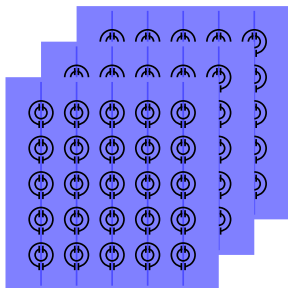
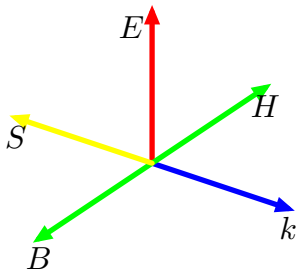
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Left-handed materials

$$k^2 = \epsilon\mu \frac{\omega^2}{c^2}$$

- If $\epsilon < 0$ y $\mu < 0$, k is real \implies propagating field.

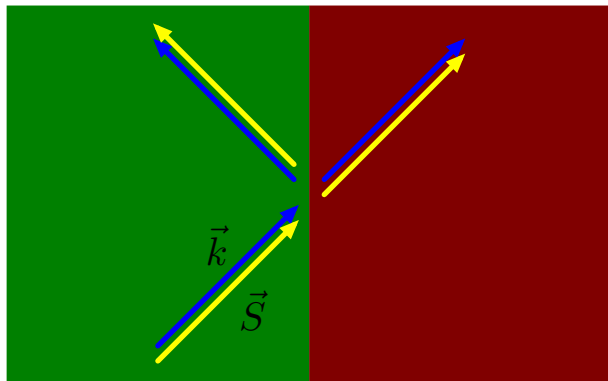


- but \mathbf{S} (energy flux) would be opposite to \mathbf{k} (phase velocity). *Group velocity* opposes *phase velocity*.



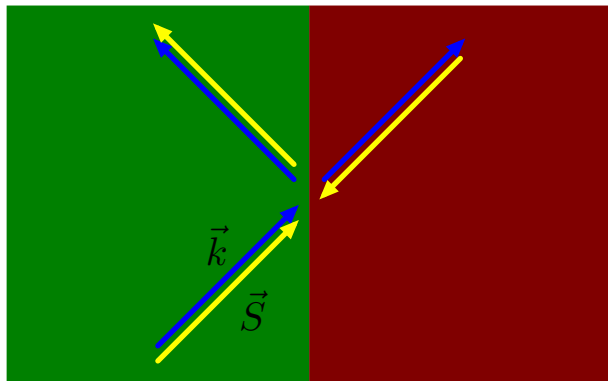
Negative refraction

$$n < 0$$



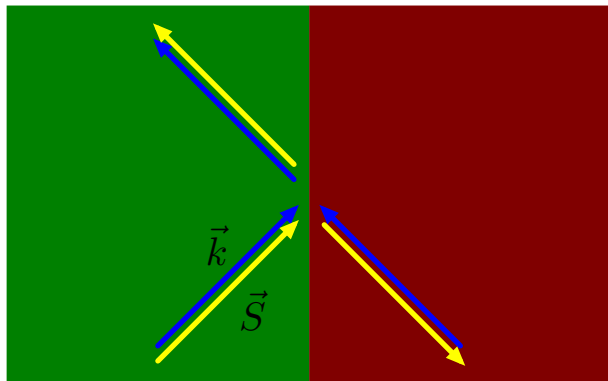
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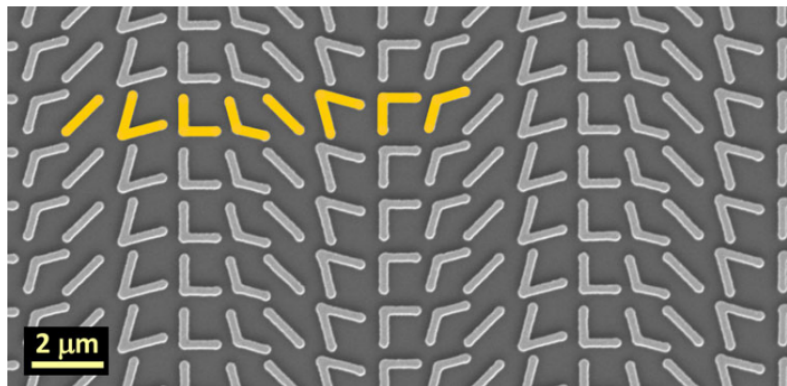


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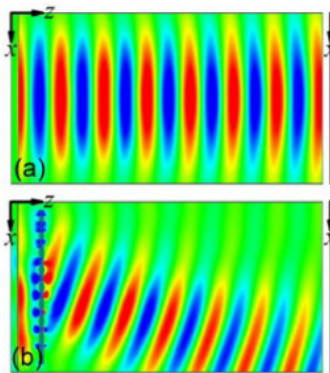
Metasurfaces



- Lateral phase controlled through geometry-dependent resonances. Controllable, polarization dependent refraction, *beyond* Snell's law.



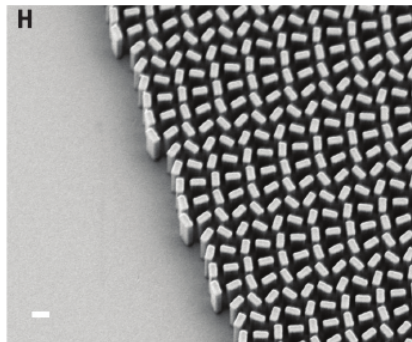
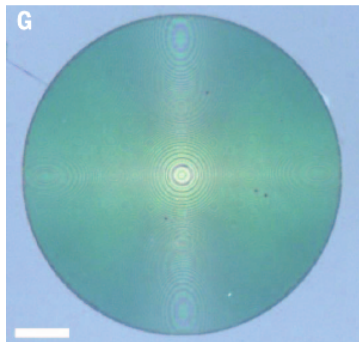
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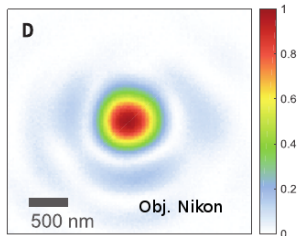
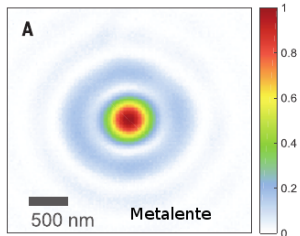
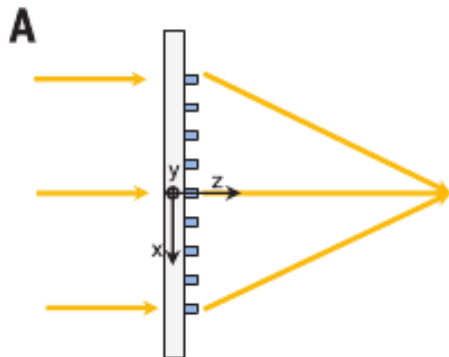
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Metalenses



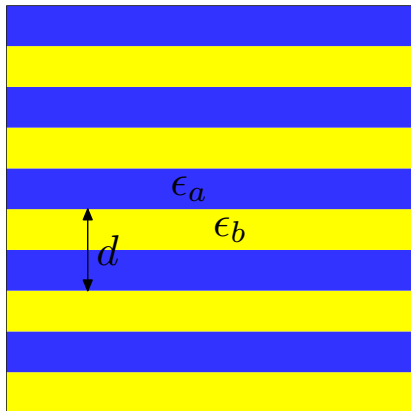
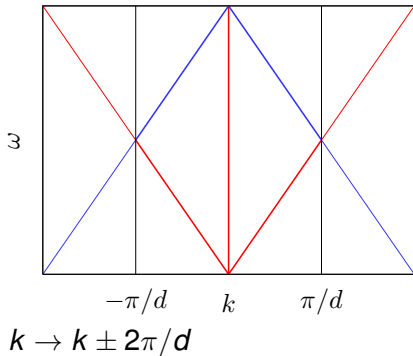
Metalenses



Photonic crystals

Superlattice

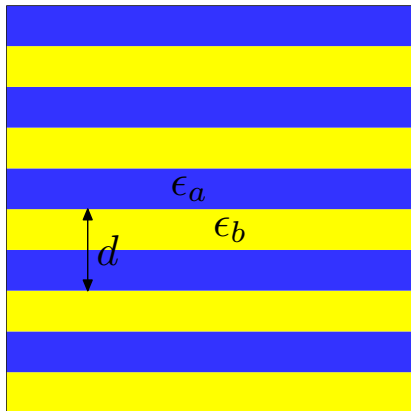
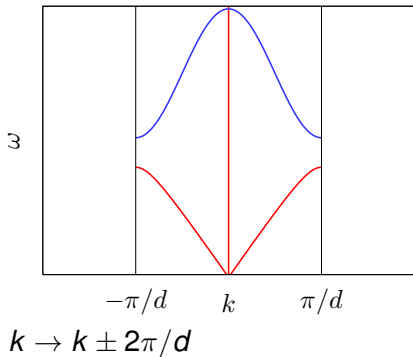
Photonic bands/gaps



Photonic crystals

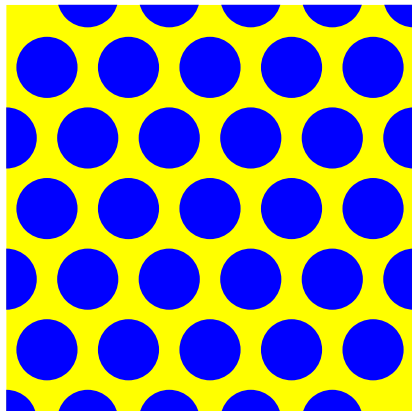
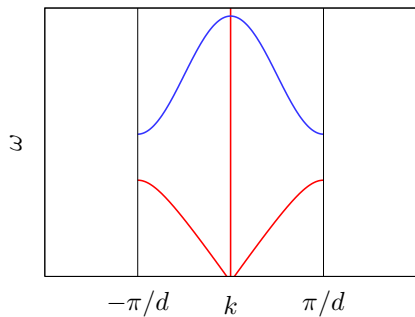
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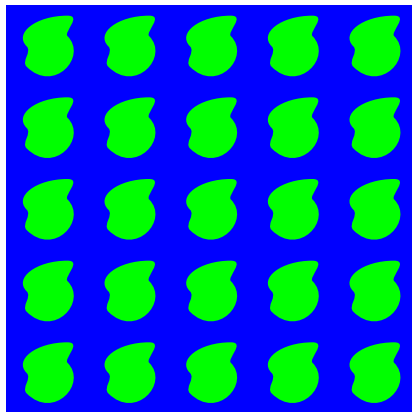
Photonic crystals

Absolute gap



The problem

- The optical properties of these materials is determined not only by their composition,
- but also by their geometry (size, shape).
- ¿How to calculate them?
- Effective response.



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Theory

Macroscopic is not any average

- $\mathbf{D} = \epsilon \mathbf{E}$
- $\langle \mathbf{D} \rangle \equiv \mathbf{D}_M = \epsilon_M \mathbf{E}_M \equiv \epsilon_M \langle \mathbf{E} \rangle$
- $\epsilon_M \neq \langle \epsilon \rangle$ as $\langle \epsilon \mathbf{E} \rangle \neq \langle \epsilon \rangle \langle \mathbf{E} \rangle$
- $\langle \epsilon \rangle$ is meaningless
- Find some operator whose average does have a meaning.



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Macroscopic response

Example: Small scale, no retardation, longitudinal response

- \mathbf{D}^L obeys the same equations with the same sources as the external longitudinal E field: $\nabla \times \mathbf{D}^L \equiv 0$,
 $\nabla \cdot \mathbf{D}^L = 4\pi\rho^{ex}$.
- \mathbf{D}^L is the external longitudinal \mathbf{E}^{ex} field.
- \mathbf{D}^L has no *spatial fluctuations* induced by the texture.
- $\mathbf{E}^L = (\epsilon^{LL})^{-1} \mathbf{D}^L$
- $\mathbf{E}_a^L = (\epsilon^{LL})_{aa}^{-1} \mathbf{D}_a^L$
- $(\epsilon_M^{LL})^{-1} = (\epsilon_{aa}^{LL})^{-1}$
- The macroscopic response to an external excitation is simply the average of the corresponding microscopic response.



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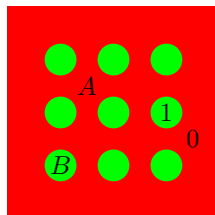
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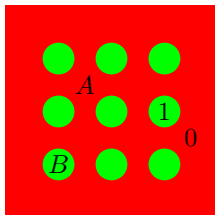
Periodic binary system

- $\epsilon(\mathbf{r}) = \begin{cases} \epsilon_B & \text{if } \mathbf{r} \in \text{inclusion,} \\ \epsilon_A & \text{if } \mathbf{r} \in \text{matrix} \end{cases}$
- Characteristic function of unit cell
$$B(\mathbf{r}) = \begin{cases} 1 & \text{if } \mathbf{r} \in \text{inclusion,} \\ 0 & \text{if } \mathbf{r} \in \text{host} \end{cases} \quad (\text{geometry})$$
- $\epsilon(\mathbf{r}) = (\epsilon_A - \epsilon_{AB}B(\mathbf{r})) = \epsilon_{AB}(u - B(\mathbf{r}))$
- $\epsilon_{AB} = \epsilon_A - \epsilon_B$
- Spectral variable $u \equiv (1 - \epsilon_B/\epsilon_A)^{-1} = \epsilon_A(\omega)/\epsilon_{AB}(\omega)$.
(composition, frequency)



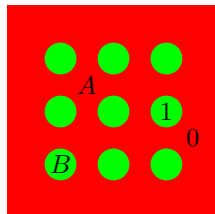
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Quantum analogy

- Periodic system, reciprocal lattice (wavevectors) $\{\mathbf{G}\}$; average: $\mathbf{G} = 0$.

- $(\epsilon_{GG'}^{LL})^{-1} \equiv [\hat{\mathbf{G}} \cdot (\epsilon_{GG'} \hat{\mathbf{G}}')]^{-1} = \frac{1}{\epsilon_{ab}} [u - B_{GG'}^{LL}]^{-1}$

- $B_{GG'}^{LL} = \hat{\mathbf{G}} \cdot B_{\mathbf{G}-\mathbf{G}'} \hat{\mathbf{G}}' \longrightarrow \hat{H}$ Hermitian operator (like a Hamiltonian in QM),
 $u \longrightarrow \epsilon$ complex 'energy',

$$\frac{\epsilon_{ab}}{\epsilon_M^{LL}} = (u - B^{LL})_{00}^{-1} / \epsilon_{ab} \longrightarrow \hat{G}_{00}(\epsilon) = \langle 0 | (\epsilon - \hat{H})^{-1} | 0 \rangle.$$

The macroscopic inverse longitudinal permittivity is analogous to a Q.M. *Green's* operator projected onto the *macroscopic state* $|0\rangle \equiv |\mathbf{G} = 0\rangle$.



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Haydock's recursion

- $|0\rangle \equiv |\mathbf{G} = 0\rangle$, $|-1\rangle \equiv 0$. Iteratively apply \hat{H} and orthogonalize:
- $\hat{H}|n-1\rangle = b_n|n\rangle + a_{n-1}|n-1\rangle + b_{n-1}|n-2\rangle \rightarrow$
Orthonormal basis $\{|n\rangle\}$ in which \hat{H} is **tridiagonal**.
- $a_{n-1} = \langle n-1|\hat{H}|n-1\rangle$
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- $\hat{H} \rightarrow B_{GG'}^{LL} = \hat{\mathbf{G}} \cdot B_{\mathbf{G}-\mathbf{G}'} \hat{\mathbf{G}}'$
- $\hat{H}|n\rangle$ is obtained by multiplying $\psi_n(\mathbf{G}) \equiv FT[\psi_n(\mathbf{r})] = \langle \mathbf{G}|n\rangle$
 - 1 by $\hat{\mathbf{G}}$ (direction of \mathbf{G} in reciprocal space,
 - 2 $B(\mathbf{r})$ in real space after IFT,
 - 3 $\hat{\mathbf{G}} \cdot$ in reciprocal space after FT,

$\rightarrow a_n$ and b_n **without matrix multiplications!**



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Theory summary

- There are *microscopic* operators \hat{O} whose *macroscopic* counterpart \hat{O}_M obeys

$$\hat{O}_M^{-1} = \langle\langle \hat{O}^{-1} \rangle\rangle = \langle 0 | \hat{O}^{-1} | 0 \rangle,$$

i.e., they are *averagable*, invert some operator and project onto *macroscopic* state.

- For example $\frac{\hat{\epsilon}^{LL}}{\epsilon_{AB}} = u - \hat{B}^{LL}$ for binary metamaterials in the nonretarded regime.
- Haydock's *recursion*: Construct basis $\{|n\rangle\}$ and Haydock coefficients a_n, b_n : apply \hat{O} , orthogonalize, normalize, repeat. . . Alternate between real and Fourier space, avoid big matrices.
- Initialize with macroscopic state $|0\rangle = |\mathbf{G} = 0\rangle$ in reciprocal (Fourier, wave-vector) space.
- $\hat{O} \rightarrow$ tridiagonal matrix with a_n in the main diagonal, b_n in the sub- and supradiagonals, and 0 elsewhere.



Continued fraction

- Macroscopic response relates to element (0,0) of the inverse of a tridiagonal symmetric matrix with elements $u - a_n, -b_n$,

$$\frac{1}{\epsilon_M^{LL}} = \frac{1}{\epsilon_{ab}} \frac{1}{u - a_0 - \frac{b_1^2}{u - a_1 - \frac{b_2^2}{u - a_2 - \frac{b_3^2}{\ddots}}}}, \quad (1)$$

- a_n, b_n depend only on geometry.
- u has all the information about composition and frequency.
- Compute a_n, b_n once, obtain ϵ_M for many compositions and frequencies substituting $u = u(\omega) = 1/(1 - \epsilon_B(\omega)/\epsilon_A(\omega))$.
- Allows for dispersion and dissipation. Useful for dielectrics and/or metals.



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Extensions

- Retardation. Lengthscale of structure comparable to wavelength of light (or even larger).
- More than two components.
- Anisotropic components.
- All of the above.
- Obtain permittivity, temporal and spatial dispersion, electromagnetic fields, photonic bands, Green's functions, reflection amplitudes, optical activity (chirality), permeability, non-linear fields, non-linear response...



Example

- Full spectra as expensive as a single value.
- 2D binary case runs in a few seconds in one core of an old (12 years) laptop.
- Easily parallelizable using `PDL::ParallelCPU` (`set_autothread_targ` and `set_autothread_size`).
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Example

- Full spectra as expensive as a single value.
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Classes, Roles

- Join pieces to build programas (choose Geometry; Haydock; dielectric Tensor, Field, Green's function. ...).
- Obtain `Geometry` from characteristic function, 2D pixelated image, dielectric function, dielectric tensor: `Photonic::Geometry::FromB`, `::FromImage2D`, `::FromEpsilon`, `::FromEpsilonTensor` consume the `Photonic::Roles::Geometry` role.
- Calculate Haydock coefficients from the nonretarded, longitudinal permittivity ϵ of binary system, from the wave equation of retarded multicomponent system using spinor representation, etc. `Photonic::LE::NR2::Haydock`, `::WE::S::Haydock`... consume the `Photonic::Roles::Haydock` role.
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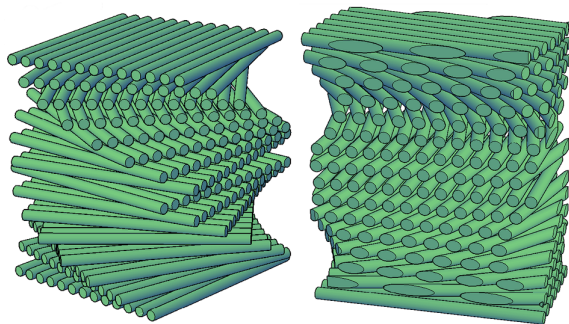
Application example



- Bouligand structure
- Bands, forbidden gap for circularly polarized light of one helicity → strong metallic like reflections, circularly polarized.



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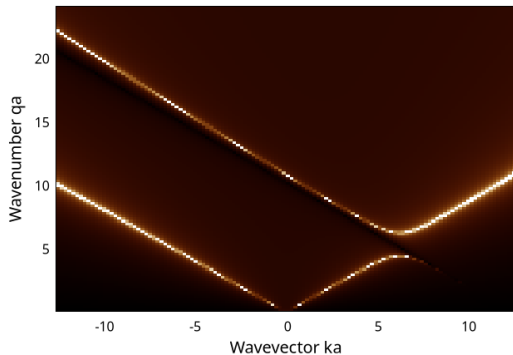


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Application example

Green's function $\epsilon_{ff}=2.00$ Pol=(1,-i)



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Conclusions

- Recursive procedures based on Haydock's representation.
- Maxwell's equations translated to Hermitian or symmetric operators that act efficiently alternating between real and reciprocal (Fourier) space.
- Macroscopic response, temporal and spatial dispersion, spectra, photonic bands, microscopic fields.
- Metamaterials with arbitrary composition and geometry. Dielectric, metals, dispersive, dissipative. . .
- Implemented in open/free computational package **Photonic**
- using **Perl** with **Moo** objects (classes, roles) and the **Perl Data Language** PDL packages for efficient number crunching.

W. Luis Mochán, Guillermo P. Ortiz, doi:10.48550/arXiv.2309.11632

